

THE PUZZLE OF EMPTY BOTTLE IN QUANTUM THEORY

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ABSTRACT. We discuss an extremely simple effect of 'shadowing' where the very existence of the measuring apparatus deforms the evolution of quantum states even if the measurement is never preformed. In spite of strange intuitive aspects, it might be related to some recent doubts about the completeness of quantum theories.

1. INTRODUCTION

While the quantum paradoxes owe their origin to the Schrödinger's thought [1] (questioning the consistency between the evolution and measurement laws), the question whether the quantum measurements consist in sudden jumps [2] or some microobject instability [3], or just in the decoherence [4] awake less attention. The last decades evolved differently, with interest focused on non-locality problems, starting from the historical Einstein–Rosen–Podolski (EPR) result [5]. Yet, the subsequent discussions [6, 7, 8, 9, 10, 11], illustrate indeed that many unsolved problems return persistently, still without final conclusions. The surprising consequence EPR [5] was the teleportation [18, 19], as well as uneasy problems with the Wheeler's paradox of "delayed choice experiment" at the bottom [12]. No less unexpected effects were described by Elitzur and Vaidman [14, 15, 16], permitting to "see in the dark" [20].

Worse, since an evidence is also accumulating that the formalism of quantum theories is not so universal as the several generations believed.

2. LINEAR NAVIGATION?

The fact which decided the form of the present quantum theories was the observation of the interference patterns of particle beams suggesting the picture of the linearly propagating de Broglie waves which after diffracting on the material obstacles paint the interference fringes (but see the discussions between de Broglie and Einstein [13]) The subsequent generalization leads to an abstract scheme of the wave-particle duality in which the (pure) microparticle states are represented by vectors in linear *Hilbert* spaces, obeying the *superposition principle*, evolving always according to a certain linear law, the process which can be interrupted by the statistical measurements, with probabilities defined by the quadratic forms of the state vectors.

A persistent desire to deal with the linear propagation (except of measurements) determined also the description of multiparticle states. In fact, any equation can be linearized at the cost of multiplying the number of variables. In case of many particles this was achieved by introducing the *entangled states* represented by vectors of the tensor product spaces, in which the basic propagation was again linear and probabilities given again by the quadratic forms, or alternatively, by

the *observables* represented by self-adjoint operators. The resulting picture, modulo postponed interpretational problems, for a long time seemed a universal form of quantum theories. The deeper troubles were always implicit in Wheeler's "delayed choice experiments", tolerated usually as a pintoresque (or even surrealistic) anecdote, far away from the typical physicist work. Yet, the difficulty is not smaller in the *interaction free measurement* [14] invading as deeply our intuitions. Here, it seems essential that the final *to be or not to be* effect happens for each single photon. However, what is the *single photon*?

To give to the dilemma even more extreme character, we permit ourselves to imagine the same EV experiment with a pair of parallel fibers quite long (see Fig.1. I beg the reader to forgive me this element of S/F story; we all know that there are no interstellar fibers!) However, assume, two of them are quite long. The next, (much shorter) part of the trajectory leads to the second beam splitter. An alternative trajectory has also its longer part: they both meet in the second splitter, opening the way to either the detector \mathbf{D}_1 or \mathbf{D}_2 . Now, if there is no obstacle in one of the trajectories, then the photon state would split into two coherent, but weaker components which would finally join at the second splitter (happily) reconstructing the initial photon state, falling into \mathbf{D}_1 .

All this is rather easy to imagine if the photon is a very short pulse. However, can the single photon propagate just as a pulse? Or perhaps, one should imagine rather as a very long, narrow wave divided by the first splitter into a pair of weaker but very long components which laboriously and slowly reconstruct their initial form at the second splitter, falling then (slowly) to the detector \mathbf{D}_1 ? If so, the problem would arise, at which moment the detector responds to our *single photon*? At the beginning or at the end of the process?

Worse, because if one of the (EV) trajectories is blocked by the bomb, then after what time the bomb explodes, but if not, then after what time the (long but incomplete) photon trajectory which *would cross the bomb* is mysteriously annihilated and contributes (again mysteriously), to the other weak component creating the propagating (complete) 1 photon state, which however arrive to another detector? We can only conclude that the story is incomplete: indeed, it is impossible to form any mental picture of the sacred *linear propagation* of the photon before the experiment is finished. And when finished, it can give an information about an obstacle which exists precisely in the place where the photon never was! Here let me remind the point made by Sudbery [29]:

"It is often stated that however puzzling some of its features may be, quantum mechanics does constitute a well defined algorithm for calculating physical quantities. Unless some form of continuous projection postulate is included as a part of the algorithm, this is not true."

A question of course remains: what is the photon wave function in the optical fiber? can it be a kind of extensive creature, many kilometers long, crawling along the fiber? It turns out that the description of the photon waves in the optical fibers is already known from the paper of I. Bialynicki-Birula [17] described not by any plane wave, but by a Bessel function cf. eq (55) (which seems a significant progress comparing to the questionable visions of Quarks as the plane waves running inside of the nucleon surfaces!)

However, except of the propagation in single fibers this does not solve the mystery of the propagation of the photon waves in [14] and indeed, it cannot by any local theory, although the final result gives an important information (about the obstacle). A sequence of studies on the imperfect cases of EV bombis recently undertaken [21, 22, 23, 24], and at least one suggests that the experiments with linearly propagating entangled states must affect the past.

In what follows, our aim is to forget for a moment about the locality problems, returning to the "historical roots", where some consequences of the Schrödinger's paradox still wait to be explored.

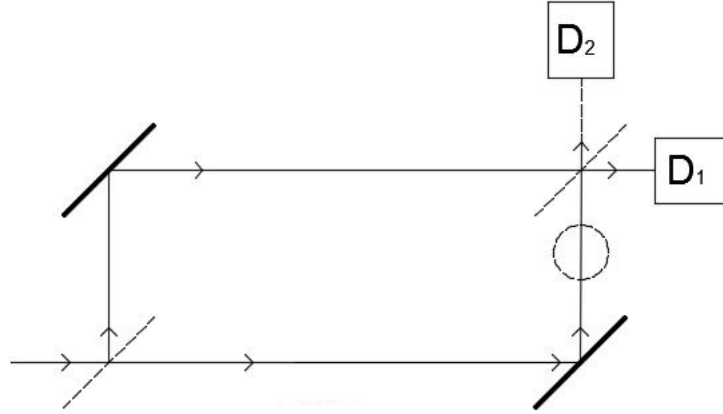


Fig 1

3. HALF FULL, HALF EMPTY.

Our story concerns a quantum system in a superposed energy state, which will be reduced – though not when the experimentalist decides, but when the system itself decides by emitting a photon (compare with the 'time of arrival' [25, 26, 27]). As a simplified model, we consider a bottle containing an atom in a state *superposed* of two lowest energy eigenstates, ground state ϕ_0 and an excited state ϕ_1 . If the atom is in the excited state, we shall say that "the bottle is full", but if in the ground state, "the bottle is empty".

In some distant past, the experimentalists examining the spectral lines imagined an atom always in one of the energy eigenstates. Today the picture is different. The existence of the superposed (but pure) energy states is unavoidable if one takes seriously the quantum mechanical formalism. The bottle is only to assure that the atom is left in peace. The only thing it can do is to radiate, settling itself on the ground state ϕ_0 . We shall assume, that at some safe distance, there is a sensitive screen in the bottle, prepared to detect the photon, should the atom radiate.

In almost all essays on the atom radiation one can find the description of the excited states as some narrow superpositions of slightly different energy eigenstates, forming an unstable state,

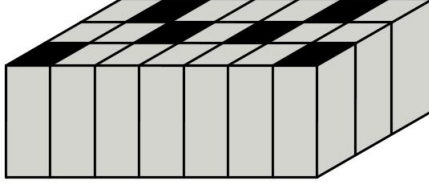


Fig 2

whose average lifetime τ is inverse to the (little) energy width ΔE , in agreement with the time-energy uncertainty, even though, the last principle awoke a lot of unfinished discussions [28]. Anyhow, by reading books on the excited state decay one can always find the considerations in which the beginning of the decay process is an excited state with a very narrow spectral line. However, I have never seen a study of a decay process starting from the superposition of two very distant energy levels. One might think that the difference is superfluous, but the excited state decay might not fulfill the principle of the linear drift, and anyhow why anybody considered the *coherent superposition* of two distant levels as the starting point of the decay process?

To fix attention, let us assume that our initial state ϕ is an equitative superposition $\phi = a_0\phi_0 + a_1\phi_1$, where $|a_0|^2 = |a_1|^2 = 1/2$ (bottle half full, half empty). From a credible phenomenology we know the behaviour of an atom in its ground state ϕ_0 . (If unperturbed, it just remains in ϕ_0 forever, $\phi_0(t) = \exp(-itE_0)\phi_0$). We know as well the behaviour of the excited state ϕ_1 . On the level of purely quantum mechanical description, this state is as stationary

$$(1) \quad \phi_1(t) = \exp(-itE_1)\phi_1$$

In reality though, the stationary evolution might be interrupted by an unpredictable photon emission with a sudden jump to the ground state ϕ_0 . For an ensemble of pure excited atoms, the number $N(t)$ of the ones surviving in the initial state ϕ_1 will be decreasing exponentially in time, $N(t) = \exp(-\gamma t)N(0)$, where $1/\gamma$ is the average life time of ϕ_1 ; the survivors landing gradually in the ground state ϕ_0 . However, what happens in a process with atoms in an initial superposed energy state $\phi = a_0\phi_0 + a_1\phi_1$ (the bottle *neither full, nor empty*)? At the first sight, it may seem that there is hardly any problem here: the evolution of the atom must simply obey the standard law

$$(2) \quad \phi(t) = a_0 e^{-iE_0 t} \phi_0 + a_1 e^{-iE_1 t} \phi_1$$

granted by the superposition principle, except if it shall suddenly radiate emitting a photon of energy $E_1 - E_0$, collocating itself on the ground state ϕ_0 . We shall show, however, that this plausible picture contains certain mysteries.

Suppose that in some atoms suddenly happens a spontaneous (*introspective*?) state reduction. They find themselves in the excited state ϕ_1 and hence, can emit the photon of the energy $E_1 - E_0$. However, average energy of the superposed initial state is less than $E_1 - E_0$. The question arises, whether they must ask some energy credit from their detector? If so, is it some influence of the detector due to its very existence, even if the measurement is not performed [30, 19], or a kind of

shadowing [32]?. Of course, not all *semi-excited* atoms can radiate. But even if the total energy balance is not violated, the single atom behavior has still some mystery.

4. THE EFFECT OF VANISHING HOPE...

It may be amazing to imagine a population of N atoms in the initial state $\phi = a_0\phi_0 + a_1\phi_1$. Assume now a gedanken scenario, in which every atom is closed in its own bottle, in form of a little, mesoscopic cell. Assume moreover, that the top surface of the cell is simultaneously a detector, sensitive to the photons of the particular energy $\hbar\omega = E_1 - E_0$. If an atom radiates, the top of its cell turns black (it is burned!). By calculating the (increasing) number of the black cells, we know how many atoms have already radiated (Fig.2). If all atoms are initially in an identical superposed state $\phi = a_0\phi_0 + a_1\phi_1$, then if somebody performed a check at the very beginning, he would find 50% of them in the ground state ϕ_0 , and henceforth, unable to radiate. However, if no initial test was performed at $t = 0$, then anyhow 50% of the atoms will never radiate. Thus, for $t \rightarrow +\infty$ all atoms must end up in the ground state ϕ_0 , though for different reasons: 1/2 of them, since they have radiated and settled down in ϕ_0 ; the remaining 1/2, just because the long waiting was equivalent to a yes-no measurement (i.e., to ask whether the atom was able at all to radiate), the negative answer reducing the ϕ_1 component to nonexistence, even though no photon was emitted. As the matter of fact, what has caused the state collapse in this last case, was not any active external interence, but just the vanishing hope (that the atom could have been in the excited state ϕ_1). Indeed, supposing that the average lifetime of the atom in the excited state ϕ_1 , e.g., is 1min , but the atom in the initial state ϕ did not radiate for 100 years, then, we can be certain that it will radiate never. According to quite orthodox statistical interpretation, this certainty means that the atom state can no longer correspond to ϕ , but it must be practically identical to ϕ_0 .

Even if the global energy balance is not affected, and even if we disregarded the principle of not postselecting, the situation seems extremely strange. While the atoms which have radiated cause almost no problem, the ones which did'nt contain a puzzle! Their superposed energy state vanished, giving place to ϕ_0 . The only external factor was our vanishing hope (take it as a rhetoric figure if you dislike!), or perhaps shadowing [32]? Anyhow, the bottle was *half full, half empty*, nothing escaped, yet the bottle is *empty*! Note that all difficulties would vanish if we simply assumed that no coherent superposition of two distant bound states can exist (remember Einstein boxes [13]?) Yet it would be too risky to derive from these alegoric remarks too premature conclusions. Anyhow, they seem to show that our theories contain some elements which are only our mental constructs: why they sometimes help (de Broglie waves!) and sometimes not, we still ignore.

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